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# On stability of nonthermal states in strongly coupled gauge theories

Alex Buchel[1,2,4] and Michael Buchel[3]

[1] *Department of Applied Mathematics,* [2] *Department of Physics and Astronomy,*
[3] *Faculty of Engineering*
*University of Western Ontario*
*London, Ontario N6A 5B7, Canada*
[4] *Perimeter Institute for Theoretical Physics*
*Waterloo, Ontario N2J 2W9, Canada*



**Abstract**

Low-energy thermal equilibrium states of strongly coupled $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM) theory on a three-sphere are unstable with respect to fluctuations breaking the global $SO(6)$ R-symmetry. Using the gauge theory/gravity correspondence, a large class of initial conditions in the R-symmetry singlet sector of the theory was been identified that fail to thermalize [1,2]. A toy model realization of such states is provided by *boson stars*, a stationary gravitational configurations supported by a complex scalar field in $AdS_5$-gravity. Motivated by the SYM example, we extend the boson star toy model to include the global $SO(6)$ R-symmetry. We show that sufficient light boson stars in the R-symmetry singlet sector are stable with respect to linearized fluctuations. As the mass of the boson star increases, they do suffer tachyonic instability associated with their localization on $S^5$. This is opposite to the behaviour of small black holes (dual to equilibrium states of $\mathcal{N} = 4$ SYM) in global $AdS_5$: the latter develop tachyonic instability as they become sufficiently light. Based on analogy with light boson stars, we expect that the R-symmetry singlet nonthermal states in strongly coupled gauge theories, represented by the quasiperiodic solutions of [2], are stable with respect to linearized fluctuations breaking the R-symmetry.

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## Contents



# 1 Introduction and summary

Consider maximally supersymmetric $\mathcal{N} = 4$ $SU(N)$ supersymmetric Yang-Mills theory on a three-sphere $S^3$ in the planar limit ($N \to \infty$, $g_{YM}^2 \to 0$ with $g_{YM}^2 N$ kept constant) and at large 't Hooft coupling, $g_{YM}^2 N \gg 1$. In this limit the theory is best described by its holographic dual [3, 4] — type IIB supergravity in asymptotically $AdS^5 \times S^5$ spacetime. The global symmetry of the five-sphere $S^5$ geometrizes the R-symmetry of the SYM. To simplify the discussion, we focus on $S^3$-invariant initial configurations of the SYM, and their evolution, consistently described within supergravity approximation. The vacuum of the theory has a Casimir energy

$$E_{vacuum} = \frac{3(N^2 - 1)}{16L}\,, \tag{1.1}$$

where $L$ is the radius of the $S^3$. An initial state with the energy

$$E = (1 + \epsilon)\ \times\ E_{vacuum} > E_{vacuum}\,, \tag{1.2}$$

if it equilibrates in the future, is described by a Schwarzschild black hole in $AdS_5 \times S^5$, with the entropy $S$, the temperature $T$, and the size[1] $r_+$ given by

$$S(\epsilon) = \frac{\pi N^2}{2^{3/2}} \left(\sqrt{1+\epsilon} - 1\right)^{3/2}\,,\ (TL)^2 = \frac{1}{2\pi^2}\,\frac{1+\epsilon}{\sqrt{1+\epsilon} - 1}\,,\ \left(\frac{r_+}{L}\right)^2 = \frac{1}{2}\left(\sqrt{1+\epsilon} - 1\right)\,. \tag{1.3}$$

[1] The size is defined as a radius of the $S^3$ measured at the black hole horizon.

The dynamical equilibration of a large class of such SYM states, with an additional assumption that the dynamics occurs in the R-symmetry singlet sector[2], has been discussed in [5, 6]. Once a black hole becomes sufficiently small (light),

$$\frac{r_+}{L} \lesssim 0.440234 \qquad \Longrightarrow \qquad \epsilon \le \epsilon_{crit} \simeq 2.53616\,, \tag{1.4}$$

it suffers from the Gregory-Laflamme (GL) instability [7] towards localization on $S^5$ [8,9,11]. The latter implies, in particular, that *any* initial condition (off the equilibrium) of the SYM with the energy less than $\epsilon_{crit}$ can not equilibrate within the R-symmetry singlet sector; in other words, the R-symmetry must be spontaneously broken in the approach to thermal equilibrium for *any* sufficiently low-energy state. First attempts to construct thermally equilibrium states in $\mathcal{N} = 4$ SYM with broken R-symmetry were undertaken in [9].

Here, we point to another possibility regarding the low-energy dynamics of $\mathcal{N} = 4$ SYM. In [5] (and further extended in [10]) it was pointed out that evolutions starting with initial data close to a single mode in AdS did not collapse. The class of non-collapsing solutions was extended in [1, 2]. A typical representative of this class is a boson-star [1] or a boson-star-like [2] (quasi-periodic[3]) configuration that is characterized by a broad distribution of a scalar profile in AdS and the dynamical evolution such that the nonlinear dispersion of the scalar energy-density overcomes the focusing effects of gravity in the AdS-cavity. Since the dynamics of [1, 2] was discussed entirely in $AdS_{d+1}$, it necessarily occurs in the R-symmetry singlet sector of the holographically dual gauge theory. It is natural to expect that unlike evolutionary trajectories that end up in the thermal state, the dynamics of the nonthermal states of [1, 2, 5, 10] is consistently restricted to the R-symmetry singlet sector — these states are stable with respect to localization in the compact manifold of the full ten-dimensional gravitational dual. The reason being the widely distributed energy-density profile of the scalar fields supporting the solution that shuts-off the tachyonic instabilities observed for small black holes in $AdS_5 \times S^5$. In this paper we present some evidence that such a scenario is indeed realized. Thus, we are led to conjecture that there is a large class of nonthermal low-energy states in strongly coupled gauge theories with unbroken R-symmetry.

The rest of the paper is organized as follows. In the next section we describe a toy model of asymptotically $AdS_5 \times S^5$ holographic correspondence which supports boson

[2]The full symmetry of the dynamics is thus $SO(4) \times SO(6)$.

[3]A recent paper [12] argues that these solutions are anchors of the AdS-stability islands.

stars. We construct boson star solutions first in the effective five-dimensional description, and further generalize the model to a ten-dimensional setting, where the compact manifold is a five-sphere and the effective five-dimensional negative cosmological constant is produced by the self-dual five-form flux. In section 3 we study stability of the ground state boson stars with respect to the linearized fluctuations breaking the $SO(6)$ symmetry of the five-sphere. We find that boson stars are indeed free from the tachyonic instabilities, provided they are light enough. We emphasize that this is *opposite* to the fate of smeared small black holes in $AdS_5$ — we expect that all states in $\mathcal{N}=4$ SYM with vanishingly small energy and unbroken $R$-symmetry are non-equilibrium. We conclude in section 4.

## 2 Boson stars in a holographic toy model

Boson stars are stationary gravitational solutions supported by a complex scalar field stress-energy tensor. In asymptotically AdS space-times they were originally discussed in [13]. We begin with boson stars in asymptotically $AdS_5$ space-time supported by a massless complex scalar field, and then extend the model to a ten-dimensional setting.

### 2.1 Five-dimensional perspective

Consider an effective action[4]

$$S_5=\frac{1}{16\pi G_N}\int_{\mathcal{M}_5} d^5\xi\sqrt{-g}\left(R_5+12-3\partial\phi\partial\bar{\phi}\right)\,, \tag{2.1}$$

where $G_N$ is a five-dimensional Newton's constant, $\phi=\phi_1+i\phi_2$ is a complex scalar field, and

$$\mathcal{M}_5=\partial\mathcal{M}_5\times\mathcal{I}\,,\qquad \partial\mathcal{M}_5=R_t\times S^3\,,\qquad \mathcal{I}=\{y\in[0,1]\}\ . \tag{2.2}$$

Adopting the line element as

$$ds_5^2=\frac{1}{y}\left(-ae^{-2\delta}dt^2+\frac{dy^2}{4y(1-y)a}+(1-y)d\Omega_3^2\right)\,, \tag{2.3}$$

where $d\Omega_3^2$ is the metric of unit radius $S^3$, and $a(y)$ and $\delta(y)$ are scalar functions of the radial coordinate $y$ describing the metric, and further assuming that the complex scalar field varies harmonically

$$\phi_1(y,t)+i\phi_2(y,t)=p(y)e^{i\omega t}\,, \tag{2.4}$$

[4]We set the curvature radius of the asymptotically $AdS_5$ solution to $L=1$.

the equations of motion describing the boson star form the following system of ODEs:

$$
\begin{aligned}
0 =&p'' + \frac{(2y-1)a - y + 2}{y(y-1)a}\ p' - \frac{\omega^2 e^{2\delta}}{4y(y-1)a^2}\ p\,,\\
0 =&\delta' + 2y(y-1)(p')^2 - \frac{e^{2\delta}p^2\omega^2}{2a^2}\,,\\
0 =&a' + 2y(y-1)(p')^2 a + \frac{(2-y)(a-1)}{y(y-1)} - \frac{e^{2\delta}p^2\omega^2}{2a}\,.
\end{aligned}
\tag{2.5}
$$

A physically relevant solutions to (2.5) must satisfy:

- asymptotically at the AdS boundary, *i.e.*, $y \to 0_+$:

$$
\begin{aligned}
p =&p_0\ y^2 + \left(\frac{4}{3}p_0 - \frac{1}{12}\omega^2 p_0\right)\ y^3 + \mathcal{O}(y^4)\,,\\
a =&1 + a_2\ y^2 + a_2\ y^3 + \mathcal{O}(y^4)\,,\\
\delta =&= 2p_0\ y^4 + \mathcal{O}(y^5)\,;
\end{aligned}
\tag{2.6}
$$

- at the origin of AdS, *i.e.*, $z = 1 - y \to 0_+$:

$$
\begin{aligned}
p =&p_0^h - \frac{1}{8}p_0^h\omega^2(d_0^h)^2\ z + \mathcal{O}(z^2)\,,\\
a =&1 - \frac{1}{4}(d_0^h)^2(p_0^h)^2\omega^2\ z + \mathcal{O}(z^2)\,,\\
\delta =&= \ln d_0^h - \frac{1}{2}(d_0^h)^2(p_0^h)^2\omega^2\ z + \mathcal{O}(z^2)\,.
\end{aligned}
\tag{2.7}
$$

We compute the mass $M \propto E - E_{vacuum}$ of the boson star as

$$
M = \int_0^1 dy\ \frac{1-y}{y^2 a}\ \left(4y(1-y)a^2(p')^2 + \omega^2 e^{2\delta}p^2\right)\,,
\tag{2.8}
$$

and its charge $Q$

$$
\begin{aligned}
Q =& \int_{S^3} dS^3 \int_0^1 dy\sqrt{-g}J^t\,, \qquad J^\mu = ig^{\mu\nu}\left(\bar\phi\partial_\nu\phi - \phi\partial_\nu\bar\phi\right)\,,\\
Q =&2\pi^2 \int_0^1 dy\ \frac{(1-y)e^{\delta}p^2\omega}{y^2 a}\,.
\end{aligned}
\tag{2.9}
$$

Note that given $p_0$, or alternatively the charge $Q$, the numerical boson star solution is determined by 4 parameters,

$$
\{\omega\,,\ a_2\,,\ p_0^h\,,\ d_0^h\}\,,
\tag{2.10}
$$

which is precisely the order of the system of ODEs (2.5).

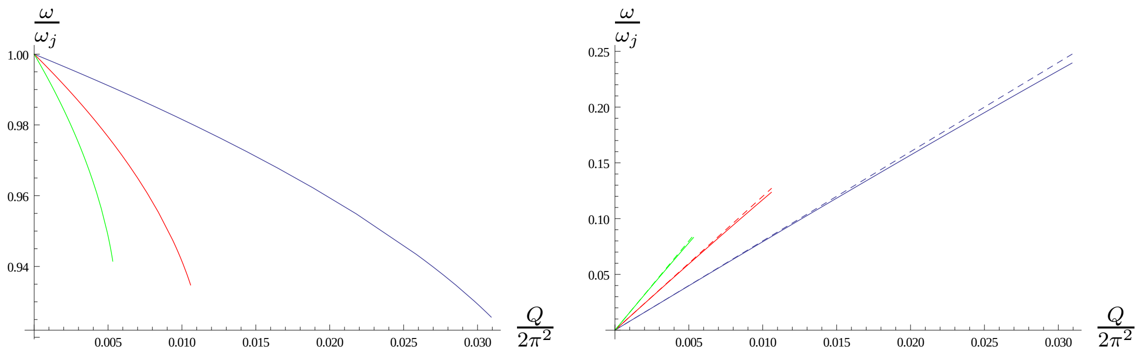


Figure 1: Frequency $\omega$ and mass $M$ of $j = \{0, 1, 2\}$ (blue,red,green) boson stars as a function of charge $Q$. The dashed lines represent perturbative relation between the mass and the charge (2.11).

The spectrum of light boson stars, *i.e.,* for $p_0 \ll 1$, is given by

$$
\begin{aligned}
& p_j = p_0 \frac{2(-1)^j}{j+2}\, y^2\, {}_2F_1\left(-j\,,\, 4+j\,;\, 2\,;\, 1-y\right) + \mathcal{O}(p_0^3)\,, \qquad \omega_j = 4 + 2j + \mathcal{O}(p_0^2)\,, \\
& a_j = 1 + \mathcal{O}(p_0^2)\,, \qquad \delta_j = \mathcal{O}(p_0)^2\,, \\
& Q_j = \frac{8\pi^2 p_0^2}{(j+2)^2((j+2)^2-1)} + \mathcal{O}(p_0^4)\,,\; M_j = \frac{4+2j}{\pi^2}\, Q_j + \mathcal{O}(Q_j^2) = \frac{\omega_j}{\pi^2}\, Q_j + \mathcal{O}(Q_j^2)\,,
\end{aligned}
\tag{2.11}
$$

where $j = 0, 1, \cdots$ is the index. For general $p_0$ (or $Q$) the boson stars can be found numerically, solving (2.5)-(2.7) with a shooting method as developed in [14]. The results of this analysis are presented in figure 1.

### 2.2 Ten-dimensional model

We would like to generalize the effective action (2.1) to a ten-dimensional setting where we would be able to study the stability of the boson star solutions of (2.1) with respect to linearized fluctuations breaking the global symmetry of the compact manifold (we choose the latter to be $S^5$). Our starting point is type IIB supergravity, where only the metric $g_{\mu\nu}^{(10)}$ and the Ramond-Ramond five-form $F_{(5)}$ are turned on. In this case the equations of motion take the form:

$$
G_{\mu\mu}^{(10)} \equiv R_{\mu\nu}^{(10)} - \frac{1}{48} F_{(5)\mu\alpha\beta\gamma\delta} F_{(5)\nu}{}^{\alpha\beta\gamma\delta} = 0\,, \qquad dF_{(5)} = 0\,, \qquad F_{(5)} = \star F_{(5)}\,. \tag{2.12}
$$

We now add a complex scalar field $\phi = \phi_1 + i\phi_2$, modifying (2.12) as follows

$$\begin{aligned} & G^{(10)}_{\mu\mu} \equiv R^{(10)}_{\mu\nu} - \frac{1}{48} F_{(5)\mu\alpha\beta\gamma\delta} F_{(5)\nu}{}^{\alpha\beta\gamma\delta} - 3\partial_\mu\phi_1\partial_\nu\phi_1 - 3\partial_\mu\phi_2\partial_\nu\phi_2 = 0\,, \\ & dF_{(5)} = 0\,, \qquad F_{(5)} = \star F_{(5)}\,. \end{aligned} \tag{2.13}$$

We use (2.13) to model ten-dimensional boson stars. The latter equations of motion are obtained in a toy model, where type IIB supergravity Lagrangian is supplemented with a complex scalar field:

$$\mathcal{L}_{IIB} \propto \int_{\mathcal{M}_{10}} d^{10}\xi \sqrt{-g_{(10)}} \biggl( R_{(10)} + \cdots \biggr) \implies \int_{\mathcal{M}_{10}} d^{10}\xi \sqrt{-g_{(10)}} \biggl( R_{(10)} - 3\partial\phi\partial\bar{\phi} + \cdots \biggr)\,. \tag{2.14}$$

We are interested in the most general ansatz describing solutions with $SO(4) \times SO(5)$ isometry[5]. To obtain an explicit expression for the equations determining such solutions we first fix the reparametrization invariance such that

$$g_{t\theta} = g_{x\theta} = 0\,. \tag{2.15}$$

We can thus write the line element as[6],

$$\begin{aligned} & ds^2_{10} = -c_1^2\ (dt)^2 + 2g_{tx}\ dtdx + c_2^2\ (dx)^2 + c_3^2\ (d\Omega_3)^2 + c_4^2\ (d\theta)^2 + c_5^2\ \sin^2\theta (d\Omega_4)^2\,, \\ & F_{(5)} = (\alpha_0\ d\theta + \alpha_1\ dt + \alpha_2\ dx) \wedge d\Omega_4 + (\alpha_3\ d\theta \wedge dt + \alpha_4\ d\theta \wedge dx + \alpha_5\ dt \wedge dx) \wedge d\Omega_3\,, \\ & c_i = c_i(t,x,\theta)\,, \qquad g_{tx} = g_{tx}(t,x,\theta)\,, \qquad \alpha_i = \alpha_i(t,x,\theta)\,, \end{aligned} \tag{2.16}$$

where $d\Omega_3$ is a volume form on a unit radius $S^3$ and $d\Omega_4$ is a volume form on a unit radius $S^4$. Next we can eliminate $\{\alpha_0, \alpha_1, \alpha_2\}$ by imposing 5-form self-duality,

$$\alpha_0 = -\frac{c_4 c_5^4 \alpha_5 \sin^4\theta}{c_3^3\sqrt{c_1^2c_2^2 + g_{tx}^2}}\,,\ \alpha_1 = -\frac{(\alpha_4 c_1^2 + \alpha_3 g_{tx}) c_5^4 \sin^4\theta}{c_4 c_3^3 \sqrt{c_1^2c_2^2 + g_{tx}^2}}\,,\ \alpha_2 = -\frac{(\alpha_3 c_2^2 - \alpha_4 g_{tx}) c_5^4 \sin^4\theta}{c_4 c_3^3 \sqrt{c_1^2c_2^2 + g_{tx}^2}}\,. \tag{2.17}$$

The resulting equations constitute a system of partial differential equations resulting from the eight non-trivial Einstein equations (the particular expressions are rather involved and we will not present their explicit form at this point):

$$G^{(10)}_{tt} = G^{(10)}_{xx} = G^{(10)}_{\Omega_3\Omega_3} = G^{(10)}_{\theta\theta} = G^{(10)}_{\Omega_4\Omega_4} = 0\,, \qquad G^{(10)}_{tx} = G^{(10)}_{t\theta} = G^{(10)}_{x\theta} = 0\,, \tag{2.18}$$

[5]We follow here the general discussion in [11].

[6]Expression for the RR five-form is the most general decomposition that preserves $SO(4) \times SO(5)$ symmetry of the ansatz.

together with the five-form equations:

$$
\begin{aligned}
0 =&\partial_\theta\alpha_3 + 4\cot(\theta)\alpha_3 - \frac{\alpha_5 c_4^2 c_1^2}{c_1^2c_2^2+g_{tx}^2}\partial_x \ln\frac{\alpha_5 c_4 c_5^4}{c_3^3\sqrt{c_1^2c_2^2+g_{tx}^2}} - \alpha_3\Bigg(\partial_\theta \ln\frac{c_1c_4c_3^3}{c_2c_5^4} \\
&+\frac{g_{tx}^2}{c_1^2c_2^2+g_{tx}^2}\partial_\theta\ln\frac{c_2}{c_1}\Bigg) - \frac{\alpha_5 g_{tx} c_4^2}{c_1^2c_2^2+g_{tx}^2}\partial_t\ln\frac{\alpha_5c_4c_5^4}{c_3^3\sqrt{c_1^2c_2^2+g_{tx}^2}} - \frac{\alpha_4c_1^4}{c_1^2c_2^2+g_{tx}^2}\partial_\theta\frac{g_{tx}}{c_1^2}\,, \\
0 =&\partial_\theta\alpha_4 + 4\cot(\theta)\alpha_4 - \frac{\alpha_5 c_2^2 c_4^2}{c_1^2c_2^2+g_{tx}^2}\partial_t \ln\frac{\alpha_5 c_4 c_5^4}{c_3^3\sqrt{c_1^2c_2^2+g_{tx}^2}} - \alpha_4\Bigg(\partial_\theta \ln\frac{c_4c_3^3}{c_5^4} \\
&+\frac{c_1^2c_2^2}{c_1^2c_2^2+g_{tx}^2}\partial_\theta\ln\frac{c_2}{c_1}\Bigg) + \frac{\alpha_5 g_{tx} c_4^2}{c_1^2c_2^2+g_{tx}^2}\partial_x\ln\frac{\alpha_5c_4c_5^4}{c_3^3\sqrt{c_1^2c_2^2+g_{tx}^2}} + \frac{\alpha_3c_2^4}{c_1^2c_2^2+g_{tx}^2}\partial_\theta\frac{g_{tx}}{c_2^2}\,, \\
0 =&\partial_t\alpha_3 - \alpha_3\partial_t\ln\frac{c_4c_3^3\sqrt{c_1^2c_2^2+g_{tx}^2}}{c_2^2c_5^4} - \frac{\alpha_4c_1^2}{c_2^2}\partial_x\ln\frac{\alpha_4c_1c_5^4}{c_2c_3^3c_4} - \frac{\alpha_3g_{tx}}{c_2^2}\partial_x\ln\frac{\alpha_3c_5^4}{c_3^3c_4} \\
&-\frac{\alpha_3g_{tx}c_1^2}{c_1^2c_2^2+g_{tx}^2}\partial_x\ln\frac{g_{tx}}{c_1c_2} + \frac{\alpha_4g_{tx}^2c_1^2}{c_2^2(c_1^2c_2^2+g_{tx}^2)}\partial_x\ln\frac{g_{tx}}{c_1c_2} - \frac{\alpha_4g_{tx}}{c_2^2}\partial_t\ln\frac{\alpha_4c_5^4}{c_3^3c_4} \\
&-\frac{\alpha_4g_{tx}c_1^2}{c_1^2c_2^2+g_{tx}^2}\partial_t\ln\frac{g_{tx}}{c_1c_2}\,, \\
0 =&\partial_\theta\alpha_5 - \partial_t\alpha_4 + \partial_x\alpha_3\,.
\end{aligned}
\tag{2.19}
$$

Boson star solutions of (2.18) and (2.19) with $SO(6)$ symmetry take the form:

$$
\begin{aligned}
&\alpha_3=\alpha_4=0\,, \qquad \alpha_5=\frac{\sin^3 x e^{-\delta(x)}}{\cos^5 x}\,, \qquad c_1=\frac{e^{-\delta(x)}\sqrt{a(x)}}{\cos x}\,, \qquad c_2=\frac{1}{\sqrt{a(x)}\cos x}\,, \\
&g_{tx}=0\,, \qquad c_3=\tan x\,, \qquad c_4=c_5=1\,, \qquad \phi=p(x)e^{i\omega t}\,,
\end{aligned}
\tag{2.20}
$$

where $\{p(x)\,,\,a(x)\,,\,\delta(x)\}$ are solutions of (2.5) with the identification

$$
\cos^2 x = y\,. \tag{2.21}
$$

## 3 Stability of level-0 boson stars

We now proceed with the stability analysis of the boson star solutions (2.20) with respect to the linearized fluctuations partially breaking the R-symmetry to $SO(5)$. To

this end we assume, to linear order in $\lambda$:

$$
\begin{aligned}
&\alpha_3 = \lambda A_3(x)\ dY_\ell(\theta)\ \cos(kt)\,, \qquad \alpha_4 = \lambda A_4(x)\ dY_\ell(\theta)\ \sin(kt)\,,\\
&\alpha_5 = \frac{\sin^3 x e^{-\delta(x)}}{\cos^5 x}\left(1+\lambda A_5(x)\ Y_\ell(\theta)\ \cos(kt)\right)\\
&g_{tx} = \lambda f(x)\ Y_\ell(\theta)\ \sin(kt)\,, \qquad c_1 = \frac{e^{-\delta(x)}\sqrt{a(x)}}{\cos x}\left(1+\lambda f_1(x)\ Y_\ell(\theta)\ \cos(kt)\right)\,,\\
&c_2 = \frac{1}{\sqrt{a(x)}\cos x}\left(1+\lambda f_2(x)\ Y_\ell(\theta)\ \cos(kt)\right)\,,\ c_3 = \tan x\left(1+\lambda f_3(x)\ Y_\ell(\theta)\ \cos(kt)\right)\,,\\
&c_4 = 1+\lambda f_4(x)\ Y_\ell(\theta)\cos(kt)\,, \qquad c_5 = 1+\lambda f_5(x)\ Y_\ell(\theta)\ \cos(kt)\,,\\
&\phi = \phi_1 + i\phi_2 = p(x)\left(1+\lambda h_1(x)\ Y_\ell(\theta)\ \cos(kt)\right)\exp\left[i\left(\omega t + \lambda h_2(x)\ Y_\ell(\theta)\ \sin(kt)\right)\right]\,,
\end{aligned}
\tag{3.1}
$$

where $Y_\ell$ are the $S^5$-spherical harmonics,

$$
\Delta_{S^5} Y_\ell \equiv -s\ Y_\ell = -\ell(\ell+4)Y_\ell\,, \qquad dY_\ell = \partial_\theta Y_\ell\,. \tag{3.2}
$$

To order $\mathcal{O}(\lambda)$, the 5-form equations (2.19) are solved with

$$
A_3 = A_4 = A_5 = 0\,, \qquad f_4 = f_5 = 0\,, \qquad f_3 = -\frac{1}{3}f_1 - \frac{1}{3}f_2\,. \tag{3.3}
$$

Next, substituting (3.1) (with (3.3) and using the radial coordinate $y$, see (2.21) ) into (2.18) we find at $\mathcal{O}(\lambda)$:

$$
\begin{aligned}
0 &= h_1' + \frac{p^2 e^{2\delta}\omega^2 y(y-1) - 4(p')^2 a^2 y^2 (y-1)^2 + 2a(2a+y-2)}{4ya^2(y-1)p'k} p\omega h_2' \\
&- \frac{1}{48p'a^4py^2(1-y)^2}\Big(-16y^4a^4(1-y)^4(p')^4 + 8a^2y^2(1-y)^2(p^2e^{2\delta}\omega^2 y(y-1) \\
&+ 2a(5a+y-2))(p')^2 + 4ay(-1+y)(-2a\omega^2p^2 - \omega^2p^2y + 2\omega^2p^2 + ak^2)e^{2\delta} \\
&- \omega^4p^4y^2(1-y)^2e^{4\delta} - 4a^2(a\ell^2(y-1) + 4a\ell(y-1) + a^2 + 6ay + y^2 - 8a - 4y + 4)\Big) f_2 \\
&- \frac{1}{48(1-y)^{3/2}y^{3/2}a^3pp'k}\Big(4a^2y^2(1-y)^2(6p^2e^{2\delta}\omega^2y - a\ell^2 - 4a\ell)(p')^2 + ay(\ell^2\omega^2p^2y \\
&- \ell^2\omega^2p^2 + 4\ell\omega^2p^2y - 24a\omega^2p^2 - 4\ell\omega^2p^2 - 12\omega^2p^2y + 24\omega^2p^2 + 6ak^2)e^{2\delta} \\
&+ 6p^4y^2\omega^4(1-y)e^{4\delta} + 2a^2\ell(\ell+4)(2a+y-2)\Big) f + \frac{1}{4ya^2(1-y)p'p}\Big(p^2e^{2\delta}\omega^2 \\
&- 4y^2pa^2(1-y)^2(p')^3 + 4a^2y(1-y)(p')^2 + (p^3\omega^2y(y-1)e^{2\delta} - 2pa(a-y+2))p'\Big) h_1 \\
&- \frac{1}{48p'a^4py^2(1-y)^2}\Big(-16y^4a^4(1-y)^4(p')^4 + (8a^2y^3(y-1)^3\omega^2p^2e^{2\delta} \\
&+ 8a^3y^2(1-y)^2(a+2y-4))(p')^2\Big) f_1 - \frac{ph_2e^{2\delta}k\omega}{4ya^2(y-1)p'}\,,
\end{aligned}
\tag{3.4}
$$

$$
\begin{aligned}
0 &= h_2'' + \frac{2ap'y(y-1) + ap(2y-1) - py + 2p}{a(y-1)yp} h_2' - \frac{e^{2\delta}y(6p^2\omega^2 + k^2) - a\ell(\ell+4)}{4(y-1)a^2y^2} h_2 \\
&+ \frac{\omega e^{2\delta}}{4y^{1/2}pa^3(1-y)^{3/2}}\Big(-4y^2pa^2(1-y)^2(p')^2 + 4a^2y(y-1)p' + p^3\omega^2y(y-1)e^{2\delta} \\
&+ 2pa(a+y-2)\Big) f + \frac{h_1e^{2\delta}k\omega}{2a^2(1-y)y}\,,
\end{aligned}
\tag{3.5}
$$

$$
0 = f' + \frac{(2ay - a - 2y + 4)}{2ay(y-1)} f + \frac{3h_2\omega p^2 + f_1 k}{(1-y)^{1/2}y^{3/2}a}\,, \tag{3.6}
$$

$$
\begin{aligned}
0 &= f_1' + \frac{3p^2\omega}{k} h_2' - \frac{e^{2\delta}y(6p^2\omega^2 + k^2) - a\ell(\ell+4)}{4(1-y)^{1/2}y^{1/2}ak} f + \frac{1}{2a^2(y-1)y}\Big(-4(p')^2a^2(1-y)^2y^2 \\
&+ \omega^2p^2y(y-1)e^{2\delta} + 2a(a+y-2)\Big) f_2\,,
\end{aligned}
\tag{3.7}
$$

$$
\begin{aligned}
0 = f_2' &+ \frac{\omega^2 p^2 y(y-1)e^{2\delta} + 2a(2a+y-2) - 4(p')^2 a^2 y^2 (1-y)^2}{4a^2(y-1)y} f_1 + \frac{y^{1/2} e^{2\delta} k f}{4(1-y)^{1/2} a} \\
&- \frac{\omega^2 p^2 y(y-1)e^{2\delta} - 2a(3a-y+2) - 4(p')^2 a^2 y^2 (1-y^2)}{4a^2(y-1)y} f_2 - 3pp'h_1 \,.
\end{aligned}
\tag{3.8}
$$

Additionally, there are two second order equations, which are consistent with (3.4)-(3.8),

$$
\begin{aligned}
0 = f_1'' &+ \frac{4(p')^2 a^2 y^2 (1-y)^2 - \omega^2 p^2 y(y-1)e^{2\delta} - 2a(a+y-2)}{2a^2(1-y)y} f_2' + \frac{2ay-a-y+2}{ay(y-1)} f_1' \\
&- \frac{e^{2\delta} k y^{1/2} f'}{2(1-y)^{1/2} a} - \frac{e^{2\delta} y(6p^2\omega^2 - k^2) - a\ell(\ell+4)}{4(y-1)a^2y^2} f_1 + \frac{3p^2 e^{2\delta}\omega^2 y + 4a}{2(y-1)a^2y^2} f_2 + \frac{3e^{2\delta} p^2 k\omega h_2}{2a^2(y-1)y} \\
&- \frac{e^{2\delta} k (4(p')^2 a^2 y^2 (1-y)^2 - \omega^2 p^2 y(y-1)e^{2\delta} - 2a(2ay-y+2)) f}{8y^{1/2}(1-y)^{3/2} a^3} + \frac{3e^{2\delta} h_1 \omega^2 p^2}{2a^2(y-1)y} \,,
\end{aligned}
\tag{3.9}
$$

$$
\begin{aligned}
0 = f_2'' &+ \frac{e^{2\delta} k y^{1/2} f'}{2(1-y)^{1/2} a} - \frac{4(p')^2 a^2 y^2 (1-y)^2 - \omega^2 p^2 y(y-1)e^{2\delta} - 2a(2a+y-2)}{2a^2(y-1)y} f_1' \\
&- 6p'ph_1' + \frac{k e^{2\delta}(4(p')^2 a^2 y^2 (1-y)^2 - \omega^2 p^2 y(y-1)e^{2\delta} - 2a(2ay-3a-y+2)) f}{8y^{1/2}(1-y)^{3/2} a^3} \\
&+ \frac{2ay-y+2}{ay(y-1)} f_2' + \frac{e^{2\delta} k^2 y - a\ell^2 - 4a\ell - 8a}{4(y-1)a^2y^2} f_2 - 6(p')^2 h_1 \,.
\end{aligned}
\tag{3.10}
$$

Further introducing

$$
\begin{aligned}
&h_1 = k\ y^{\ell/2}\ H_1 \,, \qquad h_2 = \omega\ y^{\ell/2}\ H_2 \,, \qquad f = \sqrt{1-y}\ y^{(\ell+3)/2}\ F \,, \\
&f_1 = \frac{1}{k}\ y^{(\ell+4)/2}\ F_1 \,, \qquad f_1 = \frac{1}{k}\ y^{(\ell+6)/2}\ F_2 \,,
\end{aligned}
\tag{3.11}
$$

the spectrum of $SO(5)$-invariant fluctuations about $SO(6)$-symmetric boson stars is determined solving (3.4)-(3.8) subject to the asymptotic expansions:

- asymptotically at the AdS boundary, *i.e.*, $y \to 0_+$:

$$
\begin{aligned}
H_1 =& 1 - \frac{6H^b_{2,0}\omega^2 - 3\ell^2 - \ell\omega^2 - 8\ell + 3k^2}{12(\ell+3)}\; y + \mathcal{O}(y^2)\,,\\
H_2 =& H^b_{2,0} + \frac{3H^b_{2,0}\ell^2 + H^b_{2,0}\ell\omega^2 + 8H^b_{2,0}\ell - 3H^b_{2,0}k^2 - 6k^2}{12(\ell+3)}\; y + \mathcal{O}(y^2)\,,\\
F_1 =& -\frac{1}{2}F^b_0\ell - \frac{F^b_0(\ell^3 + 6\ell^2 - \ell k^2 + 4\ell - 2k^2)}{8(\ell+1)}\; y + \mathcal{O}(y^2)\,,\\
F_2 =& -\frac{F^b_0(\ell+k^2)}{2(\ell+1)} - \frac{1}{8(\ell+3)(\ell+1)}\; \Big(8a_2F^b_0\ell^2 + F^b_0\ell^3 + F^b_0\ell^2k^2 - 96\ell p_0^2k^2 + 8a_2F^b_0\ell\\
&+ 10F^b_0\ell^2 + 7F^b_0\ell k^2 - F^b_0k^4 - 96p_0^2k^2 + 24F^b_0\ell + 22F^b_0k^2\Big)\; y + \mathcal{O}(y^2)\,,\\
F =& F^b_0 + \frac{(\ell^2 + 8\ell - k^2 + 8)F^b_0}{4(\ell+1)}\; y + \mathcal{O}(y^2)\,;
\end{aligned}
\tag{3.12}
$$

- at the origin of AdS, *i.e.*, $z = 1 - y \to 0_+$:

$$
\begin{aligned}
H_1 =& H^h_{1,0} + \Big(-\frac{1}{4}(d^h_0)^2H^h_{2,0}\omega^2 + \frac{5}{16}(d^h_0)^2F^h_{1,0}\frac{\omega^2}{k^2} - \frac{1}{8}(d^h_0)^2H^h_{1,0}k^2 + \frac{1}{8}H^h_{1,0}\ell^2 + H^h_{1,0}\ell\Big)z\\
&+ \mathcal{O}(z^2)\,,
\end{aligned}
\tag{3.13}
$$

$$
\begin{aligned}
H_2 =& H^h_{2,0} + \Big(-\frac{3}{4}(d^h_0)^2H^h_{2,0}(p^h_0)^2\omega^2 - \frac{1}{8}(d^h_0)^2H^h_{2,0}k^2 + \frac{1}{8}H^h_{2,0}\ell^2 + H^h_{2,0}\ell\\
&- \frac{1}{4}(d^h_0)^2H^h_{1,0}k^2\Big)z + \mathcal{O}(z^2)\,,
\end{aligned}
\tag{3.14}
$$

$$
\begin{aligned}
F_1 =& F^h_{1,0} + \Big(\frac{3}{4}\omega^2(p^h_0)^2(d^h_0)^2H^h_{1,0}k^2 - \frac{15}{16}F^h_{1,0}(d^h_0)^2(p^h_0)^2\omega^2 - \frac{1}{8}(d^h_0)^2k^2F^h_{1,0} + \frac{1}{8}F^h_{1,0}\ell^2\\
&+ F^h_{1,0}\ell + \frac{7}{4}F^h_{1,0}\Big)z + \mathcal{O}(z^2)\,,
\end{aligned}
\tag{3.15}
$$

$$
\begin{aligned}
F_2 =& -\frac{1}{4}F^h_{1,0} + \Big(-\frac{1}{4}\omega^2(p^h_0)^2(d^h_0)^2H^h_{1,0}k^2 + \frac{1}{8}\omega^2(p^h_0)^2(d^h_0)^2H^h_{2,0}k^2\\
&+ \frac{5}{16}F^h_{1,0}(d^h_0)^2(p^h_0)^2\omega^2 + \frac{1}{16}(d^h_0)^2k^2F^h_{1,0} - \frac{1}{48}F^h_{1,0}\ell^2 - \frac{5}{24}F^h_{1,0}\ell - \frac{1}{2}F^h_{1,0}\Big)z + \mathcal{O}(z^2)\,,
\end{aligned}
\tag{3.16}
$$

$$
\begin{aligned}
F =& \frac{3}{2}H_{2,0}^h(p_0^h)^2\omega^2 + \frac{1}{2}F_{1,0}^h + \biggl(\frac{5}{4}\omega^2(p_0^h)^2H_{2,0}^h\ell + \frac{5}{12}F_{1,0}^h\ell - \frac{5}{8}H_{2,0}^h(p_0^h)^4\omega^4(d_0^h)^2 \\
& - \frac{13}{48}F_{1,0}^h(d_0^h)^2(p_0^h)^2\omega^2 - \frac{1}{24}(d_0^h)^2k^2F_{1,0}^h + \frac{1}{24}F_{1,0}^h\ell^2 + \frac{11}{12}F_{1,0}^h + 3H_{2,0}^h(p_0^h)^2\omega^2 \\
& - \frac{1}{8}\omega^2(p_0^h)^2(d_0^h)^2H_{2,0}^hk^2 + \frac{1}{8}\omega^2(p_0^h)^2H_{2,0}^h\ell^2 - \frac{1}{4}(d_0^h)^2H_{2,0}^h(p_0^h)^2\omega^4\biggr)z + \mathcal{O}(z^2)\,.
\end{aligned}
\tag{3.17}
$$

Note that without loss of generality we normalized linearized fluctuations so that $H_1\Big|_{y=0} = 1$. Furthermore, the total number of parameters characterizing the solution,

$$
\left\{k^2\,,\ H_{2,0}^b\,,\ F_0^b\,,\ H_{1,0}^h\,,\ H_{2,0}^h\,,\ F_{1,0}^h\right\}\,,
\tag{3.18}
$$

is precisely the order of the ODE system (3.4)-(3.8).

Once again, we use the shooting method developed in [14] to determine the spectrum of the linearized fluctuations. First we outline the computations for the light boson stars, *i.e.,* for $p_0 \ll 1$, and then present the results for the general boson stars. We restrict out attention to $j = 0$ (ground state) boson stars. For $j > 0$ the radial profile of a boson star $p(y)$ has $j$ nodes inside the interval $y \in (0, 1)$. This results in additional poles in the fluctuation equations (3.4) and (3.5) which renders our shooting method inapplicable[7].

### 3.1 Spectrum of linearized fluctuations of $j = 0$ light boson stars

Using (2.11), to leading order in $p_0$,

$$
p(y) \equiv p_{j=0}(y) = p_0\ y^2 + \mathcal{O}(p_0^3)\,, \qquad \omega = \omega_{j=0} = 4 + \mathcal{O}(p_0^2)\,,
\tag{3.19}
$$

and

$$
\begin{aligned}
&H_1 = H_{1,0}(y) + \mathcal{O}(p_0^2)\,, \qquad H_2 = H_{2,0}(y) + \mathcal{O}(p_0^2)\,, \\
&F = p_0^2F_{,2}(y) + \mathcal{O}(p_0^4)\,, \qquad F_1 = p_0^2F_{1,2}(y) + \mathcal{O}(p_0^4)\,, \qquad F_2 = p_0^2F_{2,2}(y) + \mathcal{O}(p_0^4)\,,
\end{aligned}
\tag{3.20}
$$

we find from (3.4)-(3.8):

$$
\begin{aligned}
0 =& H_{1,0}' + \frac{4yH_{2,0}'}{k^2(y-1)} + \frac{(\ell(\ell+4) - k^2y + y + 3)F_{2,2}}{24y^2(y-1)k^2} + \frac{(k^2-1)F_{1,2}}{24(1-y)y^2k^2} \\
& - \frac{(2\ell^2y + 8\ell y + 6k^2y)F_{,2}}{96y^3k^2(1-y)} + \frac{(\ell(y-1) - y - 1)H_{1,0}}{2y(y-1)} + \frac{(2\ell y - 2k^2y)H_{2,0}}{yk^2(y-1)}\,,
\end{aligned}
\tag{3.21}
$$

[7]Identical technical difficulties were also observed in [1]. These difficulties can be resolved assuming a more general fluctuation ansatz for $j > 0$ as in [15].

$$0 = H''_{2,0} + \frac{(\ell(y-1)+5y-3)H'_{2,0}}{y(y-1)} + \frac{(\ell^2+8\ell-k^2)H_{2,0}}{4y(y-1)} - \frac{k^2 H_{1,0}}{2y(y-1)}\,, \tag{3.22}$$

$$0 = F'_{,2} + \frac{(\ell y-\ell+4y)F_{,2}}{2y(y-1)} - \frac{F_{1,2}}{y(y-1)} - \frac{48yH_{2,0}}{y-1}\,, \tag{3.23}$$

$$0 = F'_{1,2} + 48y^2 H'_{2,0} + \frac{(\ell^2-k^2y+4\ell)F_{,2}}{4y} + \frac{(\ell+4)F_{1,2}}{2y} + F_{2,2} + 24yH_{2,0}\ell\,, \tag{3.24}$$

$$0 = F'_{2,2} + \frac{F_{1,2}}{2y(y-1)} + \frac{(2\ell(y-1)+10y-2)F_{2,2}}{4y(y-1)} + \frac{k^2F_{,2}}{4y} - 6H_{1,0}k^2\,. \tag{3.25}$$

The system of ODEs (3.21)-(3.25) can further be reduced to a single 4th-order ODE for $H_{1,0}$:

$$\begin{split}0 = H''''_{1,0} &+ \frac{2(\ell y-\ell+7y-4)}{y(y-1)}H'''_{1,0} + \frac{1}{2y^2(1-y)^2}\biggl(3\ell^2y^2 - 5\ell^2 y + 36\ell y^2 - k^2y^2 + 2\ell^2\\ &- 50\ell y + k^2 y + 94y^2 + 14\ell - 106y + 24\biggr)H''_{1,0} + \frac{1}{2y^2(1-y)^2}\biggl(\ell^3 y - \ell^3 + 15\ell^2 y - \ell k^2 y\\ &- 13\ell^2 + \ell k^2 + 60\ell y - 5k^2 y - 40\ell + 3k^2 + 50y - 30\biggr)H'_{1,0} + \frac{1}{16y^2(1-y)^2}\biggl(\ell^4 + 16\ell^3\\ &- 2\ell^2k^2 + 64\ell^2 - 16\ell k^2 + k^4 - 64k^2\biggr)H_{1,0}\,.\end{split} \tag{3.26}$$

Solving (3.26) with the boundary conditions

$$\lim_{y\to 0_+} H_{1,0}(y) = 1\,, \qquad \lim_{y\to 1_-} H_{1,0}(y) = \text{finite}\,, \tag{3.27}$$

determines the perturbative spectrum $k^2$.

Note that (3.26) with (3.27) allows for a polynomial-in-$y$ solution:

$$\begin{split}&H_{1,0}(y) = H_{1,0,\ell,n,\pm}(y) = {}_2F_1\left(-n\,,\, l+n+4\,;\, l+3\,;\, y\right)\,,\\ &k^2 = k^2_{\ell,n,-} = (\ell+2n)^2\,, \qquad k^2 = k^2_{\ell,n,+} = (\ell+8+2n)^2\,,\end{split} \tag{3.28}$$

where $n = 0, 1, \cdots$ indexes the excitation level of a (light) ground state boson star; additionally $\pm$ denotes discrete branches for a fixed $n$. Although for different branches $H_{1,0,\ell,n,-} = H_{1,0,\ell,n,+}$, the radial profiles for the remaining fluctuations do differ: *e.g.*, for $n = 1$, solving (3.22) with the appropriate boundary conditions, (3.12) and (3.14), we find for $H_{2,0} = H_{2,0,\ell,n,\pm}$,

$$H_{2,0,\ell,1,-} = \frac{\ell+2}{4} - \frac{(\ell+5)(\ell+2)}{4(\ell+3)}\,y\,, \qquad H_{2,0,\ell,1,+} = -\frac{\ell+10}{4} + \frac{(\ell+5)(\ell+10)}{4(\ell+3)}\,y\,. \tag{3.29}$$

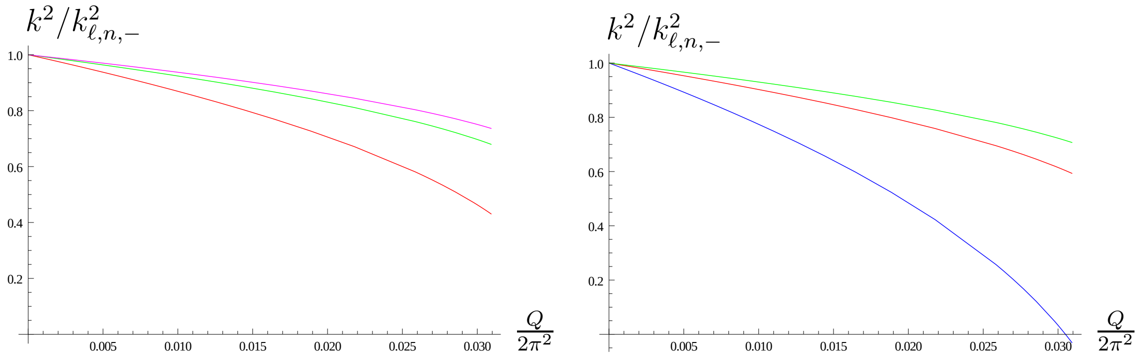


Figure 2: Low-lying states in the fluctuation spectrum of $j = 0$ boson stars as a function of charge $Q$: $\ell = \{0, 1\}$ (left panel, right panel), $n = \{0, 1, 2, 3\}$ (blue,red,green,magenta). Leading order in $Q \to 0$ frequency eigenvalues $k^2_{\ell,n,-}$ are given by (3.28).

### 3.2 Fluctuation spectrum of $j = 0$ boson stars

Perturbative in $p_0$ spectrum of the $j = 0$ boson stars (3.28) provides a starting point for the construction of the generic states labeled by $\{\ell, n, \pm\}$. These states are obtained numerically solving (3.4)-(3.8), subject to the asymptotic expansions (3.12)-(3.17). The results of the analysis are presented in figure 2. States with $\ell = 0$ represent $SO(6)$-invariant fluctuations. Furthermore, the state $\{\ell = 0, n = 0, -\}$ is a zero mode corresponding to rescaling of $\lambda$ in (3.1). We observe that the state $\{\ell = 1, n = 0, -\}$ becomes tachyonic for large values of the boson star charge $Q$ (blue line, right panel). For small values of the charge $Q$ the frequency eigenvalues of the fluctuations are close to $k^2_{\ell,n,\pm}$ (given by (3.28)) and thus are stable.

## 4 Conclusion

The low-energy dynamics of strongly coupled gauge theories in a finite volume is rather involved. Motivated by the gauge theory/string theory correspondence we discussed stability of stationary solutions, *boson stars*, supported by a complex scalar field in $AdS_5 \times S^5$. These solutions are typical representatives of $SO(4)$-invariant states in global $AdS_5$ that fail to gravitationally collapse in the limit of vanishing mass [1, 2, 5]. Unlike small smeared black holes in $AdS_5$, which are unstable with respect to localization on $S^5$ [8, 9, 11], we explicitly demonstrated that boson stars are stable with

respect to linearized perturbations leading to spontaneous breaking of the global $SO(6)$ symmetry below some critical mass (an in particular in the limit of vanishing mass). The result is far from being unexpected: the Gregory-Laflamme instability is triggered by the localization of energy in a small region of the space-time, smeared over a large compact transverse space. A characteristic feature of a boson star (and also generic solutions that fail to gravitationally collapse in global $AdS_{d+1}$) is a broad distribution of the matter profile — quite an opposite regime for the GL instability. While the GL instability excludes low-energy R-symmetry singlet equilibrium states in $\mathcal{N}=4$ SYM, our analysis suggests that there is a large class of low-energy intrinsically nonthermal states in the theory, invariant under the R-symmetry.

Our discussion was done in a toy model of the gauge theory/gravity correspondence. It would be interesting to study boson stars in type IIB supergravity proper. A starting point in this direction could be the supergravity solutions used in models of holographic superconductors, as in [16].

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